4 A company has three machines, I, II and III, each producing chocolate bar ice creams of a particular variety.

(a) Machine I produces bars whose weights are normally distributed with a mean of 48.1 grams and a standard deviation of 0.25 grams. Determine the probability that the weight of a randomly selected bar is less than 47.5 grams. *(3 marks)*

(b) Machine II produces bars whose weights are normally distributed with a standard deviation of 0.32 grams. Given also that 85 per cent of bars have weights below 50.0 grams, determine the mean weight of bars. *(4 marks)*

(c) From a random sample of 36 bars, selected at random from those produced on Machine III, calculations gave a mean weight of 52.46 grams and an unbiased estimate of the population variance of 0.1764 grams^2.

(i) Construct a 95% confidence interval for the mean weight of bars produced on Machine III, giving the limits to two decimal places. *(4 marks)*

(ii) Name the theorem that you have used and explain why it was applicable in this case. *(2 marks)*

7 (a) Machine A produces plastic balls which have diameters that are normally distributed with a mean of 10.3 cm and a standard deviation of 0.16 cm.

(i) Determine the proportion of balls which have diameters less than 10.5 cm. *(3 marks)*

(ii) Calculate the diameter exceeded by 75 per cent of balls. *(4 marks)*

(b) Machine B produces beach balls which have diameters that are normally distributed with a mean of μ cm and a standard deviation of 0.24 cm.

The diameter, d cm, of each ball in a random sample of 144 beach balls was measured. This gave:

$$\sum d = 3585.6\,.$$

(i) Calculate an unbiased estimate of μ. *(1 mark)*

(ii) Calculate the standard error of your unbiased estimate. *(2 marks)*

(iii) Construct a 95% confidence interval for μ, giving its limits to two decimal places. *(4 marks)*

(iv) Hence state, with a reason, whether you agree with a claim that $\mu = 25$. *(2 marks)*

3 Pencils produced on a certain machine have lengths, in millimetres, which are normally distributed with a mean of μ and a standard deviation of 3.

A random sample of 16 pencils was taken and the length, x millimetres, measured for each pencil, giving

$$\sum x = 2848.$$

(a) State why $\overline{X}$, the mean length, in millimetres, of a random sample of 16 pencils produced on the machine, is normally distributed. *(1 mark)*

(b) Construct a 99% confidence interval for μ. *(5 marks)*

1 The weight, in grams, of Italian grated cheese in cartons may be assumed to be normally distributed with mean μ and standard deviation 1.6.

From a random sample of 64 such cartons, the mean weight of grated cheese per carton is found to be 80.8 grams.

Construct a 95% confidence interval for μ. *(4 marks)*

5 The contents of each of a random sample of 100 cans of a soft drink are measured. The results have a mean of 331.28 ml and a standard deviation of 2.97 ml.

(a) Show that an unbiased estimate of the population variance is 8.91 ml^2. *(2 marks)*

(b) Construct a 99% confidence interval for the population mean, giving the limits to two decimal places. *(4 marks)*

(c) Explain why, in answering part (b), an assumption regarding the distribution of the contents of cans was **not** necessary. *(2 marks)*

4 (a) The volume, X millilitres, of olive oil in one-litre bottles may be assumed to be a normally distributed random variable with mean μ_X and standard deviation 3.

(i) Assuming that $\mu_X = 1005$, determine the probability that the volume of olive oil in a randomly selected bottle is less than 1010 ml. *(3 marks)*

(ii) Find, to the nearest integer, the value of μ_X so that at most 1% of bottles contain less than 1 litre of olive oil. *(4 marks)*

(b) The volume, Y millilitres, of sunflower oil in one-litre bottles may be assumed to be a normally distributed random variable with mean μ_Y and standard deviation 3.

The volume, y millilitres, of sunflower oil in each of a random sample of 16 bottles was measured, with the result that

$$\sum y = 16\,136\,.$$

Construct a 95% confidence interval for μ_Y, giving the limits to the nearest integer. *(5 marks)*

7 The volume, X millilitres, of hand cream in tubs is normally distributed with mean μ and variance σ^2.

(a) From a random sample of 50 tubs, the following information was determined, where $\bar{x}$ denotes the sample mean.

$$\sum x = 25\,065 \quad \text{and} \quad \sum (x - \bar{x})^2 = 384.16$$

(i) Construct a 99% confidence interval for μ, giving the limits to two decimal places. *(6 marks)*

(ii) State why, in answering part (a)(i), you did **not** need to use the Central Limit Theorem. *(1 mark)*

(b) It is proposed that, from a second random sample of 50 tubs, **both** a 99% confidence interval and a 90% confidence interval for μ be constructed.

State the probability that **neither** of these confidence intervals will contain μ. *(1 mark)*

(c) It is also proposed that, from a third random sample of 50 tubs, a 99% confidence interval for μ be constructed and that, from a fourth, independent, random sample of 50 tubs, a 90% confidence interval for μ be constructed.

Find the probability that **neither** of these confidence intervals will contain μ. *(2 marks)*

5 An internet service provider operates a telephone helpdesk. The random variable T denotes the duration, in minutes, of a telephone call to the helpdesk. The internet service provider recorded the duration, t minutes, of each of a random sample of 50 telephone calls.

From the recorded durations of these telephone calls, the following values were calculated, where $\bar{t}$ denotes the sample mean.

$$\sum t = 143.50 \qquad \sum (t - \bar{t})^2 = 279.8929$$

(a) Calculate an unbiased estimate of $\text{Var}(T)$ and hence estimate the standard error of $\overline{T}$. *(3 marks)*

(b) (i) Construct a 99% confidence interval for the mean duration, μ minutes, of telephone calls to the helpdesk. *(5 marks)*

(ii) Hence comment on the claim that $\mu = 3.5$. *(2 marks)*

(c) Three months later, from a random sample of 100 telephone calls, a 99% confidence interval for μ was determined correctly as (4.12, 5.38).

Indicate, giving reasons, what may now be concluded about the value of μ. *(2 marks)*

5 A machine produces steel rods with lengths that are normally distributed with mean μ and variance σ^2.

A quality control inspector uses a gauge to measure the length, x centimetres, of each rod in a random sample of 100 rods from the machine's production. The summarised data are as follows.

$$\sum x = 1040.0 \qquad \sum x^2 = 11\,102.11$$

(a) Calculate unbiased estimates of μ and σ^2. *(3 marks)*

(b) Construct a 99% confidence interval for μ. *(4 marks)*

(c) State why, in answering part (b), you did **not** need to use the Central Limit Theorem. *(1 mark)*

(d) The gauge used to measure the length is faulty. As a consequence, each measurement taken is 0.2 cm more than the true length.

Use this additional information to write down a revised confidence interval for μ. *(2 marks)*

1 Each morning, Mustafa either cycles or walks to school.

(a) The time, X minutes, taken by Mustafa to cycle to school may be modelled by a normal random variable with mean 12 and standard deviation 2.5.

Determine:

(i) $P(X < 15)$; *(2 marks)*

(ii) $P(10 < X < 15)$. *(3 marks)*

(b) On each of a random sample of 50 mornings, Mustafa records his time, y minutes, to walk to school. From his recordings, Mustafa calculates the following quantities, where $\bar{y}$ denotes the sample mean.

$$\sum y = 835.0 \qquad \sum (y - \bar{y})^2 = 533.61$$

(i) Construct a 99% confidence interval for the mean time taken by Mustafa to walk to school. *(6 marks)*

(ii) Mustafa suspects that, on average, the time he takes to walk to school is more than 25% longer than the time he takes to cycle to school.

State, with reasons, whether this suspicion is supported by your confidence interval. *(3 marks)*

Statistics 1 – Normal Distribution and Confidence Intervals
Exam Questions Pack A Mark Scheme

Question Number & Part	Solution	Marks	Total Marks	Commentary
4 (a)	$I \sim N(48.1,\ 0.25^2)$			
	$P(W < 47.5) = P\left(Z < \frac{47.5 - 48.1}{0.25}\right) =$	M1		standardising (47.45, 47.5, 47.55) (accept $\sqrt{\sigma}$, σ^2, $\mu - w$)
	$P(Z < -2.4) = 1 - P(Z < 2.4)$	m1		area change
	$= \mathbf{0.008}$	A1	(3)	awrt
(b)	$II \sim N(\mu,\ 0.32^2)$			
	$P(W < 50) = 0.85$			
	$\therefore\ P\left(Z < \frac{50 - \mu}{0.32}\right) = 0.85$	M1		standardising (49.5, 50, 50.5) (accept $\sqrt{\sigma}$, σ^2, $\mu - w$)
	$0.85 \Rightarrow z = \mathbf{1.036}$	B1		awrt 1.04
	$\therefore\ \frac{50 - \mu}{0.32} = 1.036$	m1		equating two z-values (m0 for 0.80234)
	$\therefore\ \mu = \mathbf{49.7}$	A1	(4)	awrt
(c) (i)	$95\% \Rightarrow z = \mathbf{1.96}$	B1		cao
	C I for μ is $\bar{x} \pm z \times \frac{s}{\sqrt{n}}$	M1		use of
	$\therefore\ 52.46 \pm 1.96 \times \frac{\sqrt{0.1764}}{\sqrt{36}}$	A1√		substitution, √ on z (not 0.1764 or 0.1764^2)
	$\therefore\ 52.46 \pm 0.14$			
	$\therefore$ **(52.32, 52.60)**	A1	(4)	awrt accept **(52.3, 52.6)**
(ii)	Central Limit Theorem	B1		
	Sample size sufficiently large (with reference to $\bar{X} \sim N$)	E1	(2)	E1 for this alone E0 for additional wrong reason
			[13]	

Q	Solution	Marks	Total	Comments
7 (a)	$A \sim N(10.3,\ 0.16^2)$			
(i)	$P(A<10.5)=P\left(Z<\frac{10.5-10.3}{0.16}\right)$	M1		standardising 10.45, 10.5, 10.55 with σ σ^2, $\sqrt{\sigma}$ and/or $(\mu - x)$
	$=P(Z<1.25)=$	A1		CAO $\pm$ 1.25
	0.89 (435)	A1	3	AWRT
(ii)	$P(A>a)=0.75$			
	$\Rightarrow z=-0.6745$	B1		AWFW $(\pm)0.674$ to ± 0.675 (not 0.67 or 0.68)
	$\frac{a-10.3}{0.16}=z$	M1		standardising value with σ, σ^2, $\sqrt{\sigma}$ and/or $(\mu - x)$
	$\Rightarrow \frac{a-10.3}{0.16}=-0.6745$	m1		equating two z-values (m0 for using 1- z-value)
	$a=10.2$	A1	4	AWRT
(b)(i)	$\hat{\mu}=\bar{x}=24.9$	B1	1	CAO (B0 for bias correction)
(ii)	$SE(\hat{\mu})=\frac{\sigma}{\sqrt{n}}=$	M1		use of $(\sigma, \sigma^2, \sqrt{\sigma}) \div (n, \sqrt{n})$ with n>1 (accept with 0.16, 0.16^2, $\sqrt{0.16}$) (M0 for bias correction)
	0.02 Cannot gain (i) and (ii) from (iii)	A1	2	CAO
(iii)	$95\% \Rightarrow z = 1.96$	B1		CAO
	CI for μ is: $\hat{\mu} \pm z \times SE(\hat{\mu})$	M1		use of; must have used n>1 in (ii)
	$\therefore 24.9 \pm 1.96 \times 0.02$ $\therefore 24.9 \pm 0.04$	A1√		√ on (i) and (ii)
	24.86, 24.94	A1	4	AWRT
(iv)	As 95% CI excludes 25	B1√		√ on (iii)
	Disagree with claim	E1√	2	√ on (iii); dep on B1√
	Only mark if CI of the form (a,b) with a and b determined			
	Total		**16**	

3 (a)	Length, $X \sim$ Normal	E1	1	OE; not CLT or σ known
(b)	$\hat{\mu} = \bar{x} = \frac{1}{n}\sum x = \frac{2848}{16} = 178$	B1		CAO
	CI: $\bar{x} \pm \frac{z\sigma}{\sqrt{n}}$	M1		Use of with $n>1$
	99% $\Rightarrow z = 2.5758$	B1		AWFW 2.57 to 2.58
	Thus CI is $178 \pm \frac{2.5758 \times 3}{\sqrt{16}}$	A1√		√on $\bar{x}$ and z; not on n
	Thus CI is 178 ± 1.9 or $(176, 180)$	A1	5	AWRT; dependent upon fully correct expression
	Total		**6**	

Q	Solution	Marks	Total	Comments
1	Weight $X \sim \mathrm{N}(\mu, 1.6^2)$ 95% $\Rightarrow z = 1.96$	B1		CAO
	CI for μ is : $\bar{x} \pm z \times \frac{\sigma}{\sqrt{n}}$	M1		use of; must have $\sqrt{n}$ $(n>1)$
	$\therefore$ $80.8 \pm 1.96 \times \frac{1.6}{\sqrt{64}}$	A1√		√ on z; allow 1.6^2 or $\sqrt{1.6}$ but must be $\sqrt{64}$ or 8
	$\therefore$ 80.8 ± 0.392			
	$\therefore$ $(80.4, 81.2)$	A1	4	AWRT NB attempt at changing σ or σ^2 to unbiased $\Rightarrow$at most B1 M1
	Total		**4**	

Q	Solution	Marks	Total	Comments
5 (a)	Unbiased estimate of σ^2 is given by $\frac{n \times (\text{sample standard deviation})^2}{n-1}$	M1		use of; accept omission of ^{2}
	$= \frac{100 \times 2.97^2}{99}$ or $\frac{100 \times 8.82(09)}{99}$	A1		CAO; OE
	$= 8.91$		2	AG
(b)	99 % $\Rightarrow z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI for μ is $\bar{x} \pm z \times \frac{(s \text{ or } \sigma)}{\sqrt{n}}$	M1		use of; must have $\sqrt{n}$ with $n>1$
	$\therefore 331.28 \pm 2.5758 \times \frac{\sqrt{8.91}}{\sqrt{100}}$	A1√		ft on z only
	$\therefore 331.28 \pm 0.77$			
	$\therefore (330.51, 332.05)$	A1	4	AWRT (dep on A1√)
(c)	Large sample size so mean (approximately) normally distributed using Central Limit Theorem	B1 B1	 2	accept any $n \geq 25$ normal or CLT
	Total		**8**	

Q	Solution	Marks	Total	Comments
4(a)(i)	$X \sim \text{N}(\mu_X\ 3^2)$			
	$\text{P}(X<1010) = \text{P}\left(Z<\frac{1010-1005}{3}\right) =$	M1		Standardising (1009.5, 1010 or 1010.5) with $(\sqrt{3}, 3\,or\,3^2)$ and/or (1005 – 1010)
	$\text{P}(Z<1.67) =$	A1		AWRT; ignore sign
	0.951 to 0.953	A1	3	AWFW; (0.95221)
(ii)	P(X<1000) = 1%			
	$z_{0.01} = -2.3263$	B1		AWFW 2.32 to 2.33; ignore sign
	Also $z = \frac{1000-\mu_X}{3}$	M1		Standardising 1000 with μ_X and 3 but allow ($\mu_X - 1000$)
	Thus $\frac{1000-\mu_X}{3} = -2.3263$	m1		Equating z–value to z–term; not using 0.01, 0.99 or $\lvert 1-z \rvert$
	Thus $\mu_X = 1007$	A1	4	AWRT
(b)	$\bar{y} = \frac{16136}{16} = 1008.5$	B1		CAO
	95% implies z = 1.96	B1		CAO
	CI for μ is $\bar{y} \pm z \times \frac{\sigma}{\sqrt{n}}$	M1		Use of; must have $\sqrt{n}$ with $n > 1$ M0 for attempt at using s
	Thus $1008.5 \pm 1.96 \times \frac{3}{\sqrt{16}}$	A1√		√ on $\bar{y}$ and z only
	Thus (1007, 1010)	A1dep	5	AWRT; dependent upon fully correct expression for CI
	Total		**12**	

7 (a)(i)	$\sum x = 25065 \quad \sum (x-\bar{x})^2 = 384.16$			
	Mean, $\bar{x} = \frac{25065}{50} = 501.3$	B1		CAO
	Variance, $s^2 = \frac{384.16}{49} = 7.84$ or $s = 2.8$	B1		CAO
	[$v = 7.68$ or $\sqrt{v} = 2.77$]			AWRT
	$99\% \Rightarrow z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI for μ is: $\bar{x} \pm z \times \frac{(s \text{ or } \sigma)}{\sqrt{n}}$	M1		Use of ; must have $\sqrt{n}$ with $n > 1$
	$\therefore \quad 501.3 \pm 2.5758 \times \frac{(2.8 \text{ or } \sqrt{7.84})}{\sqrt{50}}$	A1√		ft on $\bar{x}$, s and z not on n
	$\therefore \quad 501.3 \pm 1.02$			
	$\therefore$ (500.28, 502.32)	A1	6	AWRT
(ii)	Volume, X normally distributed	E1	1	
(b)	Probability = 0.01	B1	1	CAO
(c)	Probability $= 0.01 \times 0.10 =$ 0.001	M1 A1	2	Allow $(1-(0.99 \times 0.90))$ CAO
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)	$\text{Var}(T) = s^2 = \dfrac{279.8929}{49} = 5.71$	B1		awrt (5.7121)
	$\text{SE}(\bar{T}) = \sqrt{\dfrac{\text{Var}(T)}{50}}$	M1		use of
	$= 0.338$	A1	3	Awrt [**cannot** be scored in part (b)(i)]
(b)(i)	$\bar{t} = \dfrac{143.5}{50} = 2.87$	B1		cao; can be scored in part (a)
	99% implies $z = 2.5758$	B1		awfw 2.57 to 2.58
	CI for μ is: $\bar{t} \pm z \times \dfrac{(s \text{ or } \sigma)}{\sqrt{n}}$ or $\bar{t} \pm z \times \text{SE}(\bar{t})$	M1		use of; must have $\sqrt{n}$ with $n > 1$ or equivalent or $\sqrt{n}$ in $\text{SE}(\bar{t})$
	Thus: $2.87 \pm (2.5758 \times 0.338)$	A1√		√ on $\bar{t}$, z and $\text{SE}(\bar{t}) > 0$; accept $\bar{t} = 143.5$ only if clearly stated
	Thus: (2.00, 3.74)	A1	5	awrt; accept 2 dependent on ÷ by 49 in part (a) unless subsequently corrected
(ii)	Evidence to suggest that $\mu = 3.5$ as 3.5 inside CI	B1√ B1√	2	√ on part (b)(i) clearly stated; √ on part (b)(i)
(c)	Now evidence to suggest that μ has changed/increased from 3.5 (as 3.5 outside/below CI)	B1		reason not required
	Also evidence (to suggest μ has increased during three months) as CIs do not overlap	B1	2	reason required
	Total		**12**	

Q	Solution	Marks	Total	Comments
5(a)	$\hat{\mu} = \bar{x} = \frac{1}{n}\sum x = \frac{1040}{100} = 10.4$	B1		CAO
	$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$	M1		use of; or use of $\frac{n}{n-1}v$ or v
	$= \frac{1}{99}\left(11102.11 - \frac{1040^2}{100}\right) = 2.89$	A1	3	CAO (v = 2.8611) $\left(\sqrt{v} = 1.69148\right)$
(b)	$\text{CI}: \bar{x} \pm z \times \frac{s}{\sqrt{n}}$	M1		Use of with $n > 1$
	$99\% \Rightarrow z = 2.5758$	B1		AWFW 2.57 to 2.58
	$\therefore\ 10.4 \pm 2.5758 \times \frac{1.7}{\sqrt{100}}$	A1√		√ on (a) providing $\bar{x} \neq 1040$, & on z, not on n
	$\therefore\ 10.4 \pm 0.44$ i.e. (9.96, 10.8)	A1dep	4	AWRT; dependent on ÷ by 99 in part (a) unless subsequently corrected
(c)	Length, $X \sim$ Normal	E1	1	
(d)	Require to subtract 0.2 from each CL	M1		subtract/add 0.2 from/to each CL
	$\therefore$ (9.76, 10.6)	A1√	2	√ on (b); AWRT
	Total		**10**	

Q	Solution	Marks	Total	Comments
1(a)(i)	Time, $X \sim \mathrm{N}\left(12,\ 2.5^2\right)$			
	$\mathrm{P}\left(X < 15\right) = \mathrm{P}\left(Z < \frac{15-12}{2.5}\right)$	M1		standardising (14.5, 15 or 15.5) with ($\sqrt{2.5}$, 2.5 or 2.5^2) and/or $(12 - x)$
	$\mathrm{P}\left(Z < 1.2\right) = 0.885$	A1	2	AWRT $\left(0.88493\right)$
(ii)	$\mathrm{P}\left(10 < X < 15\right) = (\mathrm{i}) - \mathrm{P}\left(X < 10\right)$ $= 0.88493 - \mathrm{P}(Z < 0.8)$	M1		OE
	$= 0.88493 - \left(1\Phi\left(0.8\right)\right)$	M1		area change
	$= 0.88493 - (1 - 0.78814) = 0.673$	A1	3	AWRT (0.67307)
(b)(i)	$\bar{y} = \frac{835.0}{50} = 16.7$	B1		CAO
	$s^2 = \frac{533.61}{49} = 10.89$ or $s = 3.3$			CAO; either
	$v = \frac{533.61}{50} = 10.6722$ or $\sqrt{v} = 3.2668$	B1		AWRT 10.67 or AWRT 3.27
	$99\% \Rightarrow z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI for μ is $\bar{y} \pm z \times \frac{\left(s \text{ or } \sqrt{v}\right)}{\sqrt{n}}$	M1		use of; must have ($\div \sqrt{n}$) with $n > 1$
	Thus: $16.7 \pm 2.5758 \times \frac{\left(3.3 \text{ or } 3.27\right)}{\sqrt{50}}$	A1√		√ on $\bar{y}$, z, (s or $\sqrt{v}$); not on n
	Thus: $\left(15.5,\ 17.9\right)$	A1	6	AWRT; dependent on $\div$ 49 for variance unless subsequently corrected
(ii)	Adding 25% to 12 gives 15	B1		CAO; seen somewhere
	Since 15 is outside/below CI	E1√		√ on (b)(i); must use 15
	Mustafa's suspicion is supported	B1√	3	√ on (b)(i); must use 15
	Total		**14**	