

## Statistics 1 – Binomial Exam Questions Pack A

- 6 The probability that at least one computer is available at any time in an 'Internet Cafe' is 0.75.
- (a) Mr World makes 16 visits to the cafe. Calculate the probability that, on entry, at least one computer is available on exactly 10 occasions. *(3 marks)*
  - (b) Miss Wyde makes 30 visits to the cafe. Determine the probability that, on entry, at least one computer is available on 20 or more occasions. *(4 marks)*
- 5 The proportions of people with blood groups  $O$ ,  $A$ ,  $B$  and  $AB$  in a particular population are in the ratio  $48 : 35 : 12 : 5$ , respectively.
- (a) Determine the probability that a random sample of 20 people from the population contains:
    - (i) exactly 10 with blood group  $O$ ; *(4 marks)*
    - (ii) at most 2 with blood group  $AB$ ; *(2 marks)*
    - (iii) at least 8 with blood group  $A$ . *(3 marks)*
- 2 At a particular supermarket, 85 per cent of customers have the supermarket clubcard.
- (a) Determine the probability that, in a random sample of 5 customers, exactly 4 customers have the supermarket clubcard. *(2 marks)*
  - (b) Determine the probability that, in a random sample of 40 customers, more than 30 customers have the supermarket clubcard. *(3 marks)*
- 6 Twenty per cent of coloured beads used in costume jewellery are blue.
- (a) Determine the probability that
    - (i) in a string of 20 beads, more than 3 beads are blue, *(3 marks)*
    - (ii) in a string of 28 beads, exactly 4 beads are blue. *(3 marks)*
  - (b) State **one** assumption that you have made about the beads in answering part (a). *(1 mark)*
- 2 (a) The probability that Azher, who always walks to work, arrives late on any day is 0.11.
- Calculate the probability that, during a period of 20 working days, he is late on exactly 4 days. *(3 marks)*
- (b) The probability that Brenda, who always cycles to work, arrives late on any day is 0.15.
- Find the probability that, during a period of 50 working days, she is late on at most 10 days. *(2 marks)*

- 3 A manufacturer of balloons produces 40 per cent that are oval and 60 per cent that are round.

Packets of 20 balloons may be assumed to contain random samples of balloons. Determine the probability that such a packet contains:

- (a) an equal number of oval balloons and round balloons; (3 marks)
- (b) fewer oval balloons than round balloons. (2 marks)

A customer selects packets of 20 balloons at random from a large consignment until she finds a packet with exactly 12 round balloons.

- (c) Give a reason why a binomial distribution is **not** an appropriate model for the number of packets selected. (1 mark)

- 3 A bin contains 400 coloured erasers that fit on the ends of pencils. The number of erasers of each colour is as follows.

| Colour | Green | Blue | Red | Yellow |
|--------|-------|------|-----|--------|
| Number | 88    | 60   | 160 | 92     |

- (a) A random sample of 25 erasers is selected, **with replacement**, from the bin. Find the probability that:

- (i) exactly 2 erasers are green; (4 marks)
- (ii) at most 3 erasers are blue; (2 marks)
- (iii) between 8 erasers and 12 erasers, inclusive, are red. (4 marks)

- (b) Erasers are selected at random, **without replacement**, from the bin until 5 yellow erasers are obtained.

Give **two** reasons why a binomial distribution does **not** model the number of erasers selected. (2 marks)

- 3 A Tourist Information Office organises guided walks. An analysis shows that 68% of its customers reserve places on Walk A, 45% of its customers reserve places on Walk B, and 25% of its customers reserve places on Walk C. Reservations may be assumed to be independent.

- (a) Calculate the probability that, in a random sample of 15 customers, exactly 10 reserve places on Walk A. (3 marks)
- (b) Determine the probability that, in a random sample of 40 customers, at least 15 but at most 20 reserve places on Walk B. (3 marks)

- 6 Toothbrushes have bristles that are either firm, medium or sensitive. The proportions of toothbrushes with these bristles are 0.12, 0.53 and 0.35 respectively.

A random sample of 50 toothbrushes is selected.

- (a) Calculate the probability that the sample contains exactly 6 toothbrushes with **firm** bristles. (3 marks)
- (b) Determine the probability that the sample contains at most 20 toothbrushes with **sensitive** bristles. (2 marks)

- 3 Paper clips are produced in a variety of colours.

- (a) The proportion of red paper clips produced is 0.20.

Determine the probability that, in a random sample of 50 coloured paper clips, the number of red clips is:

- (i) fewer than 10; (3 marks)
- (ii) at least 8 but at most 12. (3 marks)

- 4 A recent large-scale survey established that 15 per cent of cars have faulty brake lights.

- (a) Calculate the probability that, in a random sample of 18 cars, exactly 2 cars have faulty brake lights. (3 marks)
- (b) Determine the probability that, in a random sample of 50 cars, more than 5 cars but fewer than 10 cars have faulty brake lights. (3 marks)

## Statistics 1 – Binomial Exam Questions Pack A Mark Scheme

|     |     |   |    |   |                             |
|-----|-----|---|----|---|-----------------------------|
| 6   | (a) | Binomial ( $n, 0.75$ )                          | B1 |   | CAO, attempted use anywhere |
|     |     | $P(X = 10) = \binom{16}{10}(0.75)^{10}(0.25)^6$ | M1 |   | Use of $B(16, 0.75)$ and 10 |
|     |     | $= 8008 \times 0.75^{10} \times 0.25^6$         |    |   |                             |
|     |     | $= 0.11$  | A1 | 3 | AWRT                        |
| (b) |     | $P(Y \geq 20 \mid n = 30, p = 0.75)$            | M1 |   | Attempted switch            |
|     |     |   |    |   | Use of normal               |
|     |     |   |    |   | MO                          |
|     |     | $P(Y' \leq 10 \mid n = 30, p = 0.25)$           | A1 |   | $Y' \leq 10$                |
|     |     |   | A1 |   | $p = 0.25$                  |
|     |     | $= 0.89 \text{ to } 0.90$                       | A1 | 4 | AWFW 1-(0.89 to 0.90)       |
|     |     |   |    |   | M1A1A1AO                    |

| Q | Solution   | Marks | Total | Comments   |
|---|--|-------|-------|--|
| 5 | (a) $O : A : B : AB = 48 : 35 : 12 : 5$  |       |       | SC: 3 or 4 correct $p$ values: only B1   |
|   | (i) Identification of binomial with $n = 20$ ,<br>stated or implied anywhere in part (a) | B1    |       |  |
|   | $p = 0.48$   | B1    |       | CAO  |
|   | $P(O = 10) = \binom{20}{10}(0.48)^{10}(1 - 0.48)^{20-10} =$                              | M1    |       | Use of binomial formula with ${}^{20}C_x$ ;<br>$p, 1 - p; x, 20 - x$ (M0 for attempted use<br>of tables or normal approximation) |
|   | $184756 \times 0.00064925 \times 0.001445551 =$  |       |       |  |
|   | $0.1734 = 0.173 \text{ to } 0.174$   | A1    | 4     | AWFW   |
|   | (ii) $p = 0.05$  | B1    |       | CAO  |
|   | $P(AB < 2) = 0.9245 = 0.924 \text{ to } 0.925$   | B1    | 2     | AWFW   |
|   | (iii) $p = 0.35$   | B1    |       | CAO  |
|   | $P(A \geq 8) = 1 - P(A \leq 7) =$  | M1    |       | Use of<br>$0.7624 \text{ seen} \Rightarrow \text{B1}$  |
|   | $1 - 0.6010 = 0.40$  | A1    | 3     | AWRT   |

| Q     | Solution  | Marks                                      | Total | Comments   |
|-------|---|--|-------|--|
| 2 (a) | $p = 0.85$<br>$n = 5$<br>$P(X=4) = \binom{5}{4} (0.85)^4 (0.15)^1 =$<br>$5 \times 0.52201 \times 0.15 = 0.391 \text{ to } 0.392$  | M1<br>A1                                   | 2     | Use of B (5 or 40, 0.85 or 0.15) in (a) or (b); may be implied AFWW<br>$(0.8352 - 0.4437 = 0.3915)$<br>M0 for normal approximation   |
| (b)   | $n = 40$<br><b>Tables</b><br>$P(X > 30) = P(X \geq 31) = P(X' \leq x')$<br>$P(X' \leq 9) =$<br>0.933<br><br><b>Calculator</b><br>$P(X > 30) = P(X = 31, 32, \dots, 40)$<br>or<br>$P(X' \leq 9) = P(X' = 9, 8, \dots, 0)$<br><br>0.933 | M1<br>A1<br>A1<br><br>(MI)<br>(AI)<br>(AI) | 3     | Change to $X'$<br>9,10 or 11; 0.970(1) or 0.988<br>AWRT; (0.9328)<br><br>9, 10 or 11 terms<br><br>At least one 3-part term correct or 0.067<br>AWRT<br>M0 for normal approximation |

| Question Number & Part | Solution  | Marks                          | Total Marks | Commentary  |
|------------------------|---|--------------------------------|-------------|---|
| 6                      | $p = 0.2$   |                                |             |   |
| (a)                    |   |                                |             |   |
| (i)                    | $n = 20$<br>$P(B > 3) =$<br>$1 - P(B \leq 3) =$<br>$1 - 0.4114 = \underline{0.589}$   | B1<br><br>M1<br><br>A1         | (3)         | B(n, 0.2) used anywhere<br><br>use of with tables or formulae<br><br>awrt   |
| (ii)                   | $n = 28$<br>$P(B = 4) =$<br>$\binom{28}{4} (0.2)^4 (0.8)^{24} =$<br>$\underline{20475} (0.0016) (0.0047224)$<br><br>$= \underline{0.155}$ | <br><br>M1<br><br>B1<br><br>A1 | (3)         | <br><br>use of (M0 for use of tables with interpolation or normal approximation)<br><br>cao, can be implied by correct final answer<br><br>awrt |
| (b)                    | Random samples  | B1                             |             |   |

| Q     | Solution  | Marks | Total | Comments  |
|-------|---|-------|-------|---|
| 2 (a) | $n = 20 \quad p = 0.11$                             |       |       |   |
|       | Identification of binomial in parts (a) or (b)      | B1    |       | stated or implied   |
|       | $P(A = 4) = \binom{20}{4} (0.11)^4 (0.89)^{20-4} =$ | M1    |       | use of binomial formula with ${}^{20}C_x; p, 1-p; x, 20-x$          |
|       | $4845 \times 0.00014641 \times 0.154967134 =$       |       |       | (M0 for attempted use of tables or normal approximation)            |
|       | $= 0.110$   | A1    | 3     | AWRT; accept 0.11 only with evidence                                |
| (b)   | $n = 50 \quad p = 0.15$                             |       |       |   |
|       | Identification of $B(50, 0.15)$                     | B1    |       | stated or implied by correct answer or by at least one correct term |
|       | $P(B \leq 10) = 0.880(1)$                           | B1    | 2     | AWRT; accept 0.88 but only from rounding of 0.880(...)              |

| Q     | Solution                                 | Marks | Total | Comments   |
|-------|--|-------|-------|--|
| 3     | $n = 20 \quad p_0 = 0.4 \quad p_R = 0.6$ |       |       |  |
| (a)   | $P(O = 10) \text{ or } P(R = 10)$        | B1    |       | identification of $B(20, 0.4 \text{ or } 0.6)$ in (a) or (b)               |
|       | $= \binom{20}{10} (0.4)^{10} (0.6)^{10}$ | M1    |       | use of formula with ${}^{20}C_r, p+q=1, \sum \text{powers} = 20$           |
|       | $184756 \times 0.000104857 \times$       |       |       | or tables ( $\leq 10 - \leq 9$ )   |
|       | $0.006046617 = 0.8725 - 0.7553$          |       |       | (M0 for normal approx)   |
|       | $= 0.117$                                | A1    | 3     | ( $0.117 \times 2 \Rightarrow B1, M1, AO$ )<br>AWRT                        |
| (b)   | $P(O < 10) = P(0 \leq 9)$                | M1    |       | attempt at $\leq 9$ (or $\geq 11$ )  |
|       | $= 0.755$                                | A1    | 2     | (M0 for normal approx)<br>AWRT   |
| (c)   | Sample size or $n$ is not fixed          | E1    | 1     | OE<br>$n = 0$ not available $\Rightarrow E1$<br>Geometric $\Rightarrow E0$ |
| Total |  |       | 6     |  |

| Q       | Solution  | Marks | Total | Comments   |
|---------|---|-------|-------|--|
| 3(a)(i) | Binomial $n = 25$   | M1    | 4     | Attempted use of in part (a)   |
|         | $p_G = \frac{88}{400} (= 0.22)$                                   | B1    |       | cao; may be implied  |
|         | $P(G = 2) = \binom{25}{2} (0.22)^2 (0.78)^{23} =$                 | M1    |       | <b>correct</b> expression for $B(25, p)$<br>( $0 < p < 1$ ) with $x = 2$         |
|         | $300 \times 0.0484 \times 0.0032974 =$<br>0.0478 to 0.048         | A1    |       | Awfw (0.0478787)<br>[watch for $(0.22)^2 = 0.048(4)$ ]                           |
|         | (ii) $p_B = \frac{60}{400} (= 0.15)$                              | B1    | 2     | cao; may be implied by correct answer  |
|         | $P(B \leq 3) = 0.4705$ to $0.4715$                                | B1    |       | Awfw (0.4711(213))   |
|         | (iii) $p_R = \frac{160}{400} (= 0.4)$                             | B1    | 4     | cao; may be implied by correct answer<br>or $\geq 1$ correct probability         |
|         | $P(8 \leq R \leq 12)$<br>$= P(R \leq 12)$                         | M1    |       | use of $\leq 12$<br>M1 for $\geq 1$ correct term<br>M2 for 5 correct terms added |
|         | $- P(R \leq 7)$<br>$= 0.8462 - 0.1536$                            | M1    |       | use of $-$ and $\leq 7$  |
|         | $= 0.692$ to $0.693$  | A1    |       | Awfw (0.6926(805))   |
|         | (b) Number of trials/events or sample size<br>or $n$ is not fixed | B1    | 2     | B0 for $n$ not constant or decreasing, etc                                       |
|         | $P(\text{success})$ or $P(Y)$ or $p$ is not constant              | B1    |       | accept trials/events are not independent<br>or are dependent                     |
| Total   |   |       | 12    |  |

| Q    | Solution   | Marks | Total | Comments                      |
|------|--|-------|-------|-------------------------------|
| 3(a) | $A \sim B(15, 0.68)$                                 |       |       | <b>binomial only</b>          |
|      | $P(A = 10) = \binom{15}{10} (p)^{10} (1-p)^5$        | M1    |       | any $p$ providing $0 < p < 1$ |
|      | $= \binom{15}{10} (0.68)^{10} (0.32)^5$              | A1    |       | fully correct expression      |
|      | $= 3003 \times 0.021139 \times 0.003355 = 0.213$     | A1    | 3     | AWRT                          |
|      | (b) $B \sim B(40, 0.45)$                             |       |       | <b>binomial only</b>          |
|      | $P(15 \leq B \leq 20) = P(B \leq 20 \text{ or } 19)$ | M1    |       |                               |
|      | $\quad \quad \quad - P(B \leq 14 \text{ or } 15)$    | A1    |       | must include minus            |
|      | $= 0.7870 - 0.1326 = 0.654 \text{ to } 0.655$        | A1    | 3     | AWFW (0.6844 / 0.2142)        |
|      | <b>OR</b>  |       |       |                               |
|      | at least 3 terms for $B(40, 0.45)$                   | (M1)  |       |                               |
|      | answer   | (A2)  |       |                               |

| Q     | Solution  | Marks | Total | Comments   |
|-------|---|-------|-------|--|
| 6 (a) | $F: 0.12 \quad M: 0.53 \quad S: 0.35$   |       |       |  |
|       | Identification of binomial with $n = 50$ , stated or implied anywhere in question | B1    |       |  |
|       | $P(F = 6) = \binom{50}{6} (0.12)^6 (1 - 0.12)^{50-6} =$                           | M1    |       | Use of binomial formula with ${}^{50}C_x; p, 1-p; 50, 50-x$ (M0 for attempted use of tables or normal approximation) |
|       | $= 15890700 \times 0.000002985 \times 0.003607759$                                |       |       |  |
|       | 0.170 to 0.172  | A1    | 3     | AWFW   |
|       | (b) $P(S \leq 20) =$  | M1    |       | identification of $\leq 20$ ; seen or implied (M0 for attempted use of normal approximation)                         |
|       | 0.813 to 0.815  | A1    | 2     | AWFW; tables give 0.8139   |



| Q | Solution                         | Marks | Total | Comments                                     |
|---|----------------------------------|-------|-------|--|
| 3 | (a) $n = 50$ and $p_R = 0.2$     |       |       |  |
|   | (i) Model applicable is Binomial | M1    |       | Use of in (i) or (ii)                        |
|   | $P(R < 10) = P(R \leq 9)$        | M1    |       | Stated or implied from tables or by formulae |
|   | $= 0.4437$                       | A1    | 3     | AWFW 0.443 to 0.444                          |
|   | (ii) $P(8 \leq R \leq 12)$       |       |       |  |
|   | $= P(R \leq 12)$                 | M1    |       | Stated or implied from tables or by formulae |
|   | $- P(R \leq 7)$                  | A1    |       |  |
|   | $= 0.8139 - 0.1904 = 0.6235$     | A1    | 3     | AWFW 0.623 to 0.624                          |

| Q    | Solution                               | Marks | Total | Comments   |
|------|--|-------|-------|--|
| 4(a) | $n = 18$ $p = 0.15$                    |       |       |  |
|      | $P(Car = 2) =$                         | M1    |       | binomial used in (a) or (b)                      |
|      | $\binom{18}{2} (0.15)^2 (0.85)^{16}$   | A1    |       | correct expression                               |
|      | $= 0.255$ to $0.256$                   | A1    | 3     | AWFW (0.2556)                                    |
|      | (b) $n = 50$ $p = 0.15$                |       |       |  |
|      | $P(5 < Car < 10) =$                    |       |       |  |
|      | $P(Car \leq 9)$                        | M1    |       | Use of $\leq 9$ or (6, 7, 8, 9)                  |
|      | $- P(Car \leq 5)$                      | M1    |       | Use of $-$ & $\leq 5$ or (4 correct terms added) |
|      | $= 0.7911 - 0.2194 = 0.571$ to $0.572$ | A1    | 3     | AWFW (0.5717)                                    |