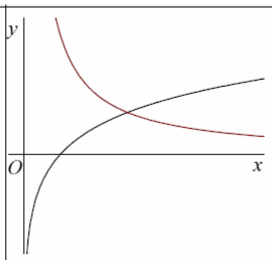
Pure Core 3 Past Paper Questions Pack B: Mark Scheme

Taken from MAP2

June 2001

Q	Solution	Marks	Total	Comments
1(a)	$3e^{2x}\cos 3x + 2e^{2x}\sin 3x$	M1 A1A1	3	Product rule
(b)	$20x\left(2x^2+1\right)^4$	M1 A1	2	for $kx\left(2x^2+1\right)^4$
	Total		**5**	

7 (a)	 Graph $\ln x$ Graph $\frac{3}{x}$	B1 B1	2	
(b)(i)	$f(3) > 0$, $f(2) < 0$ $\Rightarrow$ root in $2 < x < 3$	M1A1	2	
(ii)	$f'(x) = \frac{1}{x} + \frac{3}{x^2}$ Use of Newton-Raphson formula $x_1 = 2.82$	B1 M1A1√ A1	4	AWRT (3 s.f) is OK
	Total		**8**	

Q	Solution	Marks	Total	Comments
8 (a)	Area $= \int_0^{\pi} (x + \sin x)\,dx$	M1		
	$= \left[\frac{x^2}{2} - \cos x\right]_0^{\pi}$	A1		
	$= \left[\frac{\pi^2}{2} + 1\right] - (-1)$	M1		for correct use of limits
	$= \frac{\pi^2 + 4}{2}$ or similar	A1	4	
(b)(i)	$\int_0^{\pi} x \sin x\,dx = -x\cos x + \int \cos x\,dx$	M1A1		
	$= [-x\cos x + \sin x]_0^{\pi}$	A1√		
	$= \pi - 0$			
	$= \pi$	A1	4	AG
(ii)	$\int_0^{\pi} \sin^2 x\,dx$			
	Double angle	M1		
	$= \frac{1}{2}\int 1 - \cos 2x\,dx$	A1		
	$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$	A1√		
	$= \frac{\pi}{2}$	A1	4	AG
(c)	$V = \pi \int_0^{\pi} (x + \sin x)^2\,dx$	M1		
	$= \pi \int_0^{\pi} \left(x^2 + 2x\sin x + \sin^2 x\right)dx$			
	$= \pi\left[\frac{x^3}{3}\right]_0^{\pi} + (2\pi \times \pi) + \left(\pi \times \frac{\pi}{2}\right)$	A1√		
	$= 57.1$	A1	3	AWRT (3 s.f)
	Total		**15**	

January 2002

Q	Solution	Marks	Total	Comments
3	$y' = \frac{\cos x + (2+x)\sin x}{\cos^2 x}$	M1A1		Product rule acceptable $\frac{\cos x - (2+x)(-\sin x)}{\cos^2 x}$ M1A1 If simplified incorrectly M1A0
	$x = 0, \quad y' = 1$ $x = 0, \quad y = 2$	A1F B1		
	Tangent: $\left.\begin{array}{l}\frac{y-2}{x} = 1 \\ y = 2 + x\end{array}\right\}$	m1A1F	6	f.t. non-zero / non-infinite gradient m1 depends on first M1
	Total		**6**	

Q	Solution	Marks	Total	Comments
7 (a)	$\dfrac{(2+x)+(2-x)}{4-x^2}$	M1		
	$A=4$	A1	2	
(b)	$V=\pi\int_0^1 \dfrac{\mathrm{d}x}{4-x^2}$	M1		Condone omission of limits here
	$=\dfrac{\pi}{4}\int_0^1 \dfrac{1}{2-x}+\dfrac{1}{2+x}\mathrm{d}x$	A1F		f.t. (a) – their A
	$=\dfrac{\pi}{4}\left[-\ln\lvert 2-x\rvert+\ln\lvert 2+x\rvert\right]_0^1$	B1FB1F		Award for log integrals, ignore constant A
	$=\dfrac{\pi}{4}\left[\ln\left\lvert\dfrac{2+x}{2-x}\right\rvert\right]_0^1$	M1		Correct use of limits
	$=\dfrac{\pi}{4}\left[-\ln 3-\ln 1\right]$	A1	6	
	$=\dfrac{\pi}{4}\ln 3$			AG
(c)(i)	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=2\cos\theta$	B1		
	$\int\dfrac{2\cos\theta}{\sqrt{4-4\sin^2\theta}}(\mathrm{d}\theta)$	M1		Subs: any form ignore limits for M1/ignore omission of $\mathrm{d}\theta$
	$=\theta+c$	A1		
	$=\sin^{-1}\left(\dfrac{x}{2}\right)+c$	A1	4	
(ii)	$\text{Area}=\left[\sin^{-1}\left(\dfrac{x}{2}\right)\right]_0^1$	M1		or equivalent with θ
	$=\dfrac{\pi}{6}=\left.\begin{matrix}0.524\\0.523\\0.52\end{matrix}\right\}$	A1	2	
	Total		**14**	

June 2002

Q	Solution	Marks	Total	Comments
4	$V = \pi \int_1^2 \left(x - \frac{1}{x}\right)^2 \mathrm{d}x$	M1		for $\pi \int \left(x - \frac{1}{x}\right)^2 \mathrm{d}x$ form Condone omission of limits and dx
		A1		Correct form and limits, incl. dx (limits may be seen or implied later)
	$= \pi \int_1^2 x^2 - 2 + \frac{1}{x^2} \mathrm{d}x$	A1		for correct expansion
	$= \pi \left[\frac{x^3}{3} - 2x - \frac{1}{x}\right]_1^2$	B1√ M1		for integrating above for substitution and correct use of limits
	$= \frac{5\pi}{6}$	A1	(6)	CAO. Must be exact.
	Total		**(6)**	
5(a)(i)	$y = x \tan 3x$			
	$y' = 3x \sec^2 3x + \tan 3x$	M1A1A1	(3)	M1 for product rule. A1 each correct term
(ii)	$y = \frac{\sin x}{x}$			
	$y' = \frac{x \cos x - \sin x}{x^2}$	M1A1A1	(3)	M1 for quotient rule-ignore subsequent working A1 numerator A1 fully correct Use of product rule: $x^{-1} \cos x - x^{-2} \sin x$(or better) M1A1A1
(b)	$\int_0^{\frac{\pi}{8}} x \sin 2x \, \mathrm{d}x$			
	$= \frac{-x \cos 2x}{2} - \int \left(\frac{-\cos 2x}{2}\right) \mathrm{d}x$	M1A1A1		M1 for good attempt at 'parts' A1 each correct term
	$= \left[\frac{-x \cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{8}}$	A1√		Correctly integrating 2nd term Condone omission of limits
	$=$ use of $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$	B1		or use of $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$
	$=$ to solution (AG)	A1	(6)	
	Total		**(12)**	

January 2003

2 (a)	$I_1 = [\ln(2+u)]_0^6$ $= \ln 8 - \ln 2$ $= \ln 4$	M1 A1 A1F	3	ft if n = integer
(b)	$\mathrm{d}x = 2u\mathrm{d}u$	M1		
	$I_2 = \int_0^6 \frac{2u}{u(2+u)} \mathrm{d}u$	B1 A1		Limits Integrand
	$= 2 I_1 = 2 \ln(u+2)$	A1F		Need $I_2 = kI_1,\ k \neq 1$.
	$= \ln 16$	A1F	5	ft if m = integer
	Total		**8**	

5 (a)	Either $A\left(1, \frac{\pi}{2}\right)$ or $A(1, 90°)$	B1		**Alternative** x - coords ± 1 B1
	$B\left(-1, -\frac{\pi}{2}\right)$ or $B(-1, -90°)$	B1	2	y - coords $\pm\frac{\pi}{2}$ or $\pm$ 90° B1
(b)	Use of $x = 0.1, 0.3, 0.5, 0.7, 0.9$	M1		
	y-values: 0.1002	M1		$\sin^{-1}$(their x-values) radians
	0.3047	m1		$\sum y$ attempted (radians)
	0.5236			Accept AWRT these
	0.7754			
	1.1198			
	$I = 0.2 \times$Sum of y-values	M1		$0.2 \times \sum$ their y − values (even if degrees used)
	= 0.565	A1	5	CAO
	Total		**7**	

June 2003

Q	Solution	Marks	Total	Comments
1	$\int e^{2x}dx = \frac{1}{2}e^{2x}(+c)$			
	$\int_0^{\frac{1}{2}} x e^{2x}dx = \left[\frac{xe^{2x}}{2}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{e^{2x}}{2}dx$	M1 A1		attempt at integration by parts for $\frac{1}{2}xe^{2x}$
	$= \frac{1}{4}e - \left[\frac{e^{2x}}{4}\right]_0^{\frac{1}{2}}$	A1		for $\frac{1}{4}e^{2x}$
		m1		substitution of **both** limits attempted
	$= \frac{1}{4}e - \frac{1}{4}e + \frac{1}{4} = \frac{1}{4}$	A1	5	(CAO)
	Total		**5**	

5 (a)	$\frac{dy}{dx} = \frac{2\sin x - 2x\cos x}{\sin^2 x}$	M1		**use** of quotient rule
		A1		for numerator correct
		A1	3	for denominator correct
(b)(i)	At P, $\frac{dy}{dx} = 2$	B1F		From substitution of $x = \frac{\pi}{2}$ in answer to part (a)
	$T_P : y - \pi = 2\left(x - \frac{\pi}{2}\right)$	M1		or **use** of $y = mx + c$ to show $c = 0$ when $m = 2$
	$y = 2x$	A1	3	CAO
(ii)	Gradient of $N_P = -\frac{1}{2}$	B1F		
	$N_P : y - \pi = -\frac{1}{2}\left(x - \frac{\pi}{2}\right)$	M1		or **use** of $y = mx + c$ for their $m \neq 2$
	$y = -\frac{1}{2}x + \frac{5\pi}{4}$	A1F	3	[allow $y = -\frac{1}{2}x + 3.927$ **or** $4y + 2x = 5\pi$]
	Total		**9**	

January 2004

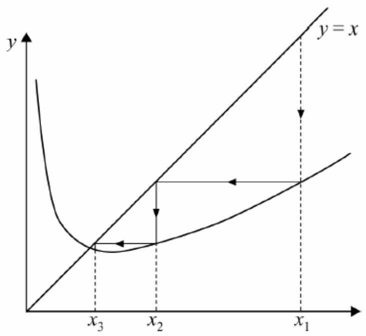
Q		Solution	Marks	Total	Comments
4	(a)	$y=\ln\left(x^2+9\right)$			
		let $u=x^2+9$ then $\frac{\mathrm{d}u}{\mathrm{d}x}=2x$ and $y=\ln u$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}u}=\frac{1}{u}=\frac{1}{x^2+9}$	M1		
		$\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x^2+9}\times 2x$	M1		Use of chain rule
		$=\frac{2x}{x^2+9}$	A1	3	CAO
	(b)	$\int_0^3 \frac{x}{x^2+9}\,\mathrm{d}x=\left[\frac{1}{2}\ln\left(x^2+9\right)\right]_0^3$	M1		
		$=\frac{1}{2}\ln 18-\frac{1}{2}\ln 9$	A1		
		$=\frac{1}{2}\ln 2$	A1	3	AG
	(c)	$\int_0^3 \frac{x+1}{x^2+9}\,\mathrm{d}x=\int_0^3 \frac{x}{x^2+9}\,\mathrm{d}x+\int_0^3 \frac{1}{x^2+9}\,\mathrm{d}x$	M1		Attempted
		$=\frac{1}{2}\ln 2+\frac{1}{3}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$	A1		
		$==\frac{1}{2}\ln 2+\frac{1}{3}\left[\tan^{-1}(-1)-\tan^{-1}(0)\right]$	M1		Limits used in correct expression
		$=\frac{1}{2}\ln 2+\frac{\pi}{12}$	A1	4	AG
		Total		**10**	

Q	Solution	Marks	Total	Comments
6 (a)	$\text{f}(1) = 0.341$ $\text{f}(2) = -0.091$ Change of sign $\Rightarrow$ $\therefore$ root in the interval $1 \le x \le 2$	M1 A1	2	
(b)(i)	$\text{f}'(x) = \cos x - \dfrac{1}{2}$	B1	1	
(ii)	$x_{n+1} = x_n - \dfrac{\text{f}(x)}{\text{f}'(x_n)} = x_n - \dfrac{\sin x_n - \frac{1}{2}x_n}{\cos x_n - \frac{1}{2}}$	M1		N-R formula used
	$x_0 = 2 \quad \therefore \quad x_1 = 2 - \dfrac{\sin 2 - 1}{\cos 2 - \frac{1}{2}}$	m1		Radians used in correct formula
	$x_1 = 1.901 \approx 1.9$	A1	3	AG
(c)(i)	$\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$ $\therefore \quad \int \sin^2 x \, \text{d}x = \dfrac{1}{2}\int (1 - \cos 2x) \, \text{d}x$ $= \dfrac{1}{2}x - \dfrac{1}{4}\sin 2x + c$	M1 A1	2	AG
(ii)	$\int_0^{1.9} \sin^2 x = \left[\dfrac{1}{2}x - \dfrac{1}{4}\sin 2x\right]_0^{1.9} = 1.10$	B1	1	
(d)	Volume of solid formed $= V_1 - V_2$	M1		
	$V_1 = \pi \int_0^{1.90} \sin^2 x \, \text{d}x$ $= \pi \times 1.10$ $(= 3.47)$	M1		for V_1 (3.46507) allow 3.46 ($1.10 \times \pi$)
	$V_2 = \dfrac{1}{3} \times \pi \times (0.95)^2 \times 1.90$ **or** $\pi \int_0^{1.9} \left(\dfrac{1}{2}x\right)^2 \text{d}x$ $(= 1.796)$	M1		for V_2
	$\therefore$ Volume of solid formed = 1.67	A1		(1.66938) allow 1.66
	Volume = 1.7 (2sf)	A1	5	
	Total		**14**	

June 2004

Q	Solution	Marks	Total	Comments
3(a)	$\int_0^{\frac{\pi}{2}} x\cos x\,\mathrm{d}x$			
	$= x\sin x - \int \sin x\,\mathrm{d}x$	M1 M1		
	$= \{x\sin x + \cos x\}_0^{\frac{\pi}{2}}$	A1		
	$= \frac{\pi}{2} - 1$	M1 A1	5	Radians only 0.570 to 0.571
(b)(i)	$t = x^2 + 4 \Rightarrow \mathrm{d}t = 2x\,\mathrm{d}x$	M1		correct
	$\therefore \int \frac{2x\,\mathrm{d}x}{\sqrt{x^2+4}} = \int \frac{\mathrm{d}t}{\sqrt{t}}$	A1	2	AG
(ii)	$\int_0^2 \frac{2x\ \mathrm{d}x}{\sqrt{x^2+4}} = \int_4^8 t^{-\frac{1}{2}}\,\mathrm{d}t$			
	$\left[2\sqrt{t}\right]$ or $\left[2\sqrt{x^2+4}\right]$	M1 A1		Integration attempted correct
	$= 2\sqrt{8} - 2\sqrt{4}$	M1		attempt at correct limits seen
	$= 2(2\sqrt{2}) - 4$			
	$= 4(\sqrt{2} - 1)$	A1	4	AG (AWRT 1.7)
	Total		**11**	

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x \times 2\cos 2x + \mathrm{e}^x \times \sin 2x$	M1 A1A1	3	Use of product rule A1 for each part correct
(ii)	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=0} = 2$			
	$\therefore y = mx \Rightarrow$ equation of tangent at (0, 0) is $y = 2x$	M1 A1ft	2	
(b)	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\pi} = 2\mathrm{e}^{\pi}$			
	$\therefore$ gradient of normal at $x=\pi$ is $-\frac{1}{2\mathrm{e}^{\pi}}$	M1		Use of $m_1 \times m_2 = -1$ (– 0.216)
	when $x = \pi,\ y = 0$	B1		
	$\therefore$ equation of normal at $(\pi, 0)$ is given by $y = -\frac{1}{2\mathrm{e}^{\pi}}(x-\pi)$	M1ft		on their gradient of normal
	$\Rightarrow 2\mathrm{e}^{\pi}y + x = \pi$	A1	4	AG (any correct form)
	Total		**9**	

Q	Solution	Marks	Total	Comments
5(a)	$f(x) = x^3 - 15$			
	$f(2) = -7 < 0$	B1		values
	$f(3) = 12 > 0$	E1	2	change of sign
	$\therefore$ root in the interval $[2, 3]$			
(b)(i)	$x = \frac{2}{3}x + \frac{5}{x^2}$			
	$(\times 3x^2) \Rightarrow 3x^3 = 2x^3 + 15$	M1		
	$x^3 - 15 = 0$	A1	2	AG
(ii)	$x_{n+1} = \frac{2}{3}x_n + \frac{5}{x_n^2}$			
	using $x_1 = 3$,	M1		
	$x_2 = 2.555556$	A1		
	$x_3 = 2.469299$	A1√		on their x_2
	$x_4 = 2.466216$	A1√	4	2.466215932
(iii)		B2	2	B1 for staircase B1 for convergence
(iv)	$\sqrt[3]{15}$	B1	1	
	Total		**11**	