General Certificate of Education
June 2009
Advanced Level Examination

**MATHEMATICS** **MPC3**
**Unit Pure Core 3**

Friday 5 June 2009 1.30 pm to 3.00 pm

**For this paper you must have:**
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** (a) The curve with equation

$$y = \frac{\cos x}{2x+1}, \qquad x > -\frac{1}{2}$$

intersects the line $y = \frac{1}{2}$ at the point where $x = \alpha$.

(i) Show that $\alpha$ lies between 0 and $\frac{\pi}{2}$. *(2 marks)*

(ii) Show that the equation $\frac{\cos x}{2x+1} = \frac{1}{2}$ can be rearranged into the form

$$x = \cos x - \frac{1}{2}$$ *(1 mark)*

(iii) Use the iteration $x_{n+1} = \cos x_n - \frac{1}{2}$ with $x_1 = 0$ to find $x_3$, giving your answer to three decimal places. *(2 marks)*

(b) (i) Given that $y = \frac{\cos x}{2x+1}$, use the quotient rule to find an expression for $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence find the gradient of the normal to the curve $y = \frac{\cos x}{2x+1}$ at the point on the curve where $x = 0$. *(2 marks)*

**2** The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x+5}, \qquad \text{for real values of } x, \; x \geqslant -2.5$$

$$\mathrm{g}(x) = \frac{1}{4x+1}, \qquad \text{for real values of } x, \; x \neq -0.25$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is $\mathrm{f}^{-1}$.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the domain of $\mathrm{f}^{-1}$. *(1 mark)*

(c) The composite function fg is denoted by h.

(i) Find an expression for $\mathrm{h}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{h}(x) = 3$. *(3 marks)*

**3** (a) Solve the equation $\tan x = -\frac{1}{3}$, giving all the values of $x$ in the interval $0 < x < 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation

$$3\sec^2 x = 5(\tan x + 1)$$

can be written in the form $3\tan^2 x - 5\tan x - 2 = 0$. *(1 mark)*

(c) Hence, or otherwise, solve the equation

$$3\sec^2 x = 5(\tan x + 1)$$

giving all the values of $x$ in the interval $0 < x < 2\pi$ in radians to two decimal places. *(4 marks)*

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**4** (a) Sketch the graph of $y = |50 - x^2|$, indicating the coordinates of the point where the graph crosses the $y$-axis. *(3 marks)*

(b) Solve the equation $|50 - x^2| = 14$. *(3 marks)*

(c) Hence, or otherwise, solve the inequality $|50 - x^2| > 14$. *(2 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 50 - x^2$. *(4 marks)*

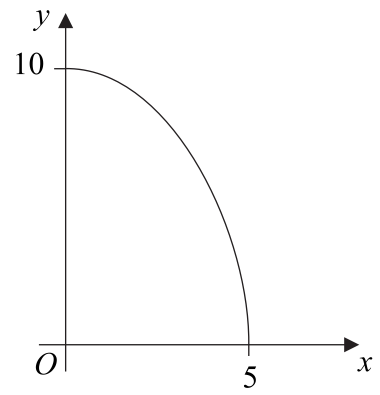
**5** (a) Given that $2 \ln x = 5$, find the exact value of $x$. *(1 mark)*

(b) Solve the equation

$$2 \ln x + \frac{15}{\ln x} = 11$$

giving your answers as exact values of $x$. *(5 marks)*

**6** The diagram shows the curve with equation $y = \sqrt{100 - 4x^2}$, where $x \geqslant 0$.




(a) Calculate the volume of the solid generated when the region bounded by the curve shown above and the coordinate axes is rotated through 360° about the ***y*-axis**, giving your answer in terms of $\pi$. *(5 marks)*

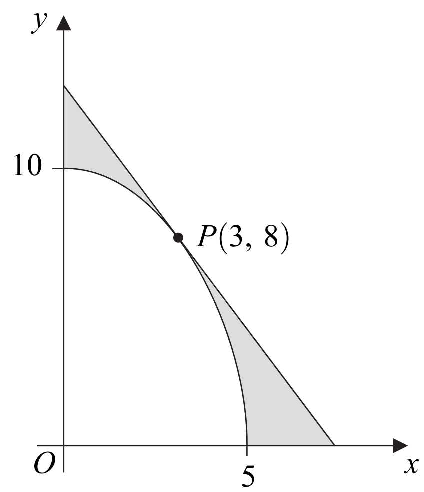
(b) Use the mid-ordinate rule with five strips of equal width to find an estimate for $\int_0^5 \sqrt{100 - 4x^2}\, dx$, giving your answer to three significant figures. *(4 marks)*

(c) The point $P$ on the curve has coordinates $(3, 8)$.

(i) Find the gradient of the curve $y = \sqrt{100 - 4x^2}$ at the point $P$. *(3 marks)*

(ii) Hence show that the equation of the tangent to the curve at the point $P$ can be written as $2y + 3x = 25$. *(2 marks)*

(d) The shaded regions on the diagram below are bounded by the curve, the tangent at $P$ and the coordinate axes.




Use your answers to part (b) and part (c)(ii) to find an approximate value for the **total** area of the shaded regions. Give your answer to three significant figures. *(5 marks)*

**7** (a) Use integration by parts to find $\int (t-1)\ln t \,\mathrm{d}t$. *(4 marks)*

(b) Use the substitution $t = 2x + 1$ to show that $\int 4x \ln(2x+1)\,\mathrm{d}x$ can be written as $\int (t-1)\ln t \,\mathrm{d}t$. *(3 marks)*

(c) Hence find the exact value of $\int_0^1 4x \ln(2x+1)\,\mathrm{d}x$. *(3 marks)*

**END OF QUESTIONS**

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