General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Thursday 17 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\dfrac{dy}{dx}$ when:

(i) $y = (2x^2 - 5x + 1)^{20}$; *(2 marks)*

(ii) $y = x\cos x$. *(2 marks)*

(b) Given that

$$y = \frac{x^3}{x-2}$$

show that

$$\frac{dy}{dx} = \frac{kx^2(x-3)}{(x-2)^2}$$

where k is a positive integer. *(3 marks)*

2 (a) Solve the equation $\cot x = 2$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(2 marks)*

(b) Show that the equation $\operatorname{cosec}^2 x = \dfrac{3\cot x + 4}{2}$ can be written as

$$2\cot^2 x - 3\cot x - 2 = 0$$ *(2 marks)*

(c) Solve the equation $\operatorname{cosec}^2 x = \dfrac{3\cot x + 4}{2}$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

3 The equation

$$x + (1 + 3x)^{\frac{1}{4}} = 0$$

has a single root, α.

(a) Show that α lies between -0.33 and -0.32. *(2 marks)*

(b) Show that the equation $x + (1 + 3x)^{\frac{1}{4}} = 0$ can be rearranged into the form

$$x = \tfrac{1}{3}(x^4 - 1)$$ *(2 marks)*

(c) Use the iteration $x_{n+1} = \dfrac{(x_n{}^4 - 1)}{3}$ with $x_1 = -0.3$ to find x_4, giving your answer to three significant figures. *(3 marks)*

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^3, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x-3}, \qquad \text{for real values of } x,\ x \neq 3$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 64$. *(3 marks)*

(c) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

5 (a) (i) Given that $y = 2x^2 - 8x + 3$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(1 mark)*

(ii) Hence, or otherwise, find

$$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\,\mathrm{d}x$$

giving your answer in the form $k \ln 3$, where k is a rational number. *(4 marks)*

(b) Use the substitution $u = 3x - 1$ to find $\int x\sqrt{3x-1}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

Turn over for the next question

6 (a) Sketch the curve with equation $y = \operatorname{cosec} x$ for $0 < x < \pi$. *(2 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.1}^{0.5} \operatorname{cosec} x \, dx$, giving your answer to three significant figures. *(4 marks)*

7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 - 5$. *(4 marks)*

(b) Sketch the graph of $y = |4x^2 - 5|$, indicating the coordinates of the point where the curve crosses the y-axis. *(3 marks)*

(c) (i) Solve the equation $|4x^2 - 5| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4x^2 - 5| \geqslant 4$. *(2 marks)*

8 (a) Given that $e^{-2x} = 3$, find the exact value of x. *(2 marks)*

(b) Use integration by parts to find $\int x e^{-2x} \, dx$. *(4 marks)*

(c) A curve has equation $y = e^{-2x} + 6x$.

(i) Find the exact values of the coordinates of the stationary point of the curve. *(4 marks)*

(ii) Determine the nature of the stationary point. *(2 marks)*

(iii) The region R is bounded by the curve $y = e^{-2x} + 6x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(5 marks)*

END OF QUESTIONS